



Structural Distortions, Macroeconomic Determinants, and Portfolio Allocation in the Maghreb Region: A Panel-Based Markowitz Mean-Variance Approach

Nawal FERHOUNE

University of Jijel (Algeria)

nawel.ferhoune@univ-jijel.dz

Atmane MEDINI

University of Eloued (Algeria)

medini-atmane@univ-eloued.dz

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Abstract:

This paper applies modern portfolio theory to the Maghreb by treating national economies as assets in a regional macro-portfolio. Using annual real GDP growth for Algeria, Tunisia, Morocco, and Libya/Mauritania over 1990–2021, we estimate panel models (pooled, fixed, random effects) and find that foreign direct investment and governance quality significantly enhance macroeconomic returns, while cross-country diversification gains are limited by very high return correlations. Mean–variance optimization via quadratic programming shows that the minimum-variance and efficient portfolios collapse to a 100% allocation to Algeria, and unconstrained models with short-selling generate extreme, economically implausible positions. However, strategic analysis reveals that this dominance is a "structural illusion" rather than genuine economic efficiency. Algeria's risk-adjusted superiority is deeply rooted in a rentier economy dependent on hydrocarbon revenues, translating this absolute allocation into severe concentration risk rather than horizontal competitiveness. Overall, the results emphasize that unlocking genuine regional diversification requires breaking rentier monopolies, enhancing the business climate, and pursuing deep institutional reforms to attract productive foreign investment.

Keywords: portfolio optimization; mean–variance; quadratic programming; panel data; efficient frontier; risk-adjusted returns.

JEL Classification: G11; D81; C61; C23; C58; G32

1.Introduction

Regional economic integration and risk-sharing mechanisms are central concerns in international macroeconomics and development finance, particularly in emerging regions. In this context, the North African region presents a structurally heterogeneous economic landscape, characterized by diverse production structures and varying exposure to commodity price shocks,



alongside notable differences in financial development levels, despite geographical proximity and historical linkages.

In Algeria and Libya, the hydrocarbon sector forms the backbone of economic activity, public revenues, and exports, making these economies highly vulnerable to energy price fluctuations and geopolitical conditions. Conversely, Morocco and Tunisia rely on more diversified production structures, with stronger industrial and service sectors, whereas Mauritania's economy is highly concentrated in the primary sector and very sensitive to external demand. These structural differences imply heterogeneous risk–return profiles across the region, potentially creating significant diversification opportunities within a regional portfolio. However, the extent to which these economies align along an efficient regional frontier remains an empirically unresolved question.

This study adopts the Markowitz approach, whose theoretical foundations in the trade-off between risk and return originate from modern portfolio theory developed by Harry Markowitz, demonstrating that rational asset allocation under uncertainty depends not only on expected returns but also on the variance–covariance structure of asset returns (Markowitz, 1952). The mean–variance framework provides a mathematical representation of efficient allocation by identifying portfolios that minimize risk for a given level of expected return or maximize return for a given level of risk, forming a cornerstone of modern portfolio theory. Although originally developed for financial asset portfolios, this framework has gradually been extended to macroeconomic aggregates, sovereign bonds, and country-level indices, allowing national economies themselves to be treated as assets within a regional portfolio. In this sense, applying the mean–variance model to national economies provides an analytical lens to assess structural diversification, co-movement, and regional risk-sharing potential.

In this context, North African economies provide a suitable case for applying the mean–variance framework at the national level, treating macroeconomic performance indicators as components of a regional portfolio. This approach allows for a more precise understanding of the potential to reduce aggregate volatility through diversification across regional economies and highlights the limits of achievable regional stability given the current growth and risk structures. Despite the growing literature on portfolio optimization and regional integration, several key gaps remain in this type of study, which this research seeks to address:

- ✓ Limited application of mean–variance analysis to economic structures, as most studies focus on financial markets or financial return indicators rather than macroeconomic fundamentals.
- ✓ Scarcity of research addressing structural balances and shared macroeconomic risks across regional economies, compared to the dominance of financial market integration studies.
- ✓ Limited use of the efficient frontier concept in explaining regional development issues, with few studies translating findings into policy recommendations on economic convergence, stabilization, or regional cooperation.



- ✓ The treatment of national macroeconomic performance indicators as components of a regional portfolio, through estimating expected returns, volatility measures, and inter-country covariance structures, and constructing an efficient frontier reflecting structural trade-offs between risk and return at the regional level.

It is noteworthy that the R programming environment was used due to its effectiveness and flexibility in modeling mean–variance portfolios and estimating efficient frontiers, making it particularly suitable for this type of applied economic and financial analysis.

2. Theoretical Foundations

The theoretical foundation of Modern Portfolio Theory (MPT), as systematically formalized for the first time by Harry Markowitz (Markowitz, 1952, 1959). Markowitz demonstrated that rational allocation of assets under uncertainty depends not only on expected returns but also on the variance covariance structure of those returns. In this framework, the decision maker aims to construct a portfolio that either maximizes expected return for a given level of risk or minimizes risk for a given level of expected return. The collection of such optimal portfolios constitutes the efficient frontier, representing the best achievable trade-offs between risk and return (Elton & Gruber, 1995).

Mathematically, for a portfolio of n assets with expected returns μ_i and variances σ^2 , and covariance σ_{ij} between assets i and j , the expected return of the portfolio is defined as:

$$E(R_p) = \sum_{i=1}^n w_i \mu_i$$

and its variance is expressed as:

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

where w_i denotes the weight of asset i in the portfolio, subject to the constraint $\sum_{i=1}^n w_i = 1$. The efficient frontier is obtained by solving the optimization problem of minimizing σ_p^2 for a given $E(R_p)$ or maximizing $E(R_p)$ for a given σ_p^2 (Fabozzi et al., 2010).

Although MPT was initially developed for financial assets, it has been extended to macro-level portfolios, treating countries as “assets” and their economic indicators as “returns” (Ang & Bekaert, 2007; Reboredo, 2012). In this macroeconomic context:

- ✓ **Expected return** (σ^2) represents indicators such as GDP growth, sovereign bond yields, or other national performance measures.
- ✓ **Risk** (σ^2) captures the volatility of these economic outcomes.
- ✓ **Covariance** (σ_{ij}) measures the degree of synchronized shocks between countries, reflecting regional interdependence.

Applying the mean–variance model to national economies provides a framework to assess structural diversification, i.e., the potential to reduce aggregate volatility by combining economies with heterogeneous risk–return profiles into a regional portfolio (Lintner, 1965). The efficient



frontier in this context represents the optimal trade-off between regional growth (return) and volatility (risk), providing a quantitative benchmark for policy and planning.

For the Maghreb region, characterized by structural heterogeneity—from hydrocarbon-dependent economies (Algeria and Libya) to diversified economies (Morocco and Tunisia) and primary-sector-focused economies (Mauritania)—the mean–variance framework offers a rigorous analytical lens to evaluate cross-country risk–return trade-offs and identify opportunities for regional diversification and stabilization (Reboredo, 2012; Ang & Bekaert, 2007).

3. Related Literature

The application of Modern Portfolio Theory (MPT) has expanded significantly beyond the realm of financial asset allocation to include macroeconomic and regional portfolios. Early studies primarily focused on financial markets, analyzing how diversification reduces risk in portfolios of stocks, bonds, and commodity indices (Elton & Gruber, 1995; Fabozzi et al., 2010). These contributions established the core principle that combining assets with heterogeneous risk–return profiles can lower portfolio volatility while maintaining or even enhancing expected returns.

Subsequent research extended the mean–variance framework to country-level and regional portfolios, treating national economies as “assets” and macroeconomic indicators as “returns.” For example, Ang and Bekaert (2007) examined cross-country financial integration and its implications for portfolio diversification, while Reboredo (2012) analyzed the co-movement between macroeconomic variables and asset returns in emerging markets. These studies show that structural differences across economies – such as sectoral composition, dependence on commodities, and the degree of financial development – can significantly influence portfolio risk and diversification potential.

In the context of emerging and developing regions, several studies have applied mean–variance models to assess the potential for regional risk sharing and stabilization. The literature highlights that economies subject to asymmetric shocks or weakly correlated growth patterns offer substantial diversification gains when analyzed jointly (Kearney & Daly, 2017; Mendoza et al., 2014). Moreover, efficient frontier analysis has increasingly been used to evaluate the optimal allocation of macroeconomic “assets” in order to mitigate regional volatility and inform policy decisions.

In other regions, such as Latin America and Asia, the mean–variance framework has been employed to construct cross-country macro portfolios and quantify regional diversification opportunities, demonstrating the usefulness of this approach as a quantitative tool for economic policy design. However, such applications remain relatively limited in the case of North Africa, despite the presence of comparable features of external fragility and heterogeneous shocks.

Despite the growing body of work on macro-level mean–variance applications, there is a noticeable gap with respect to North African economies. Most research on the region focuses on financial market integration or descriptive analyses of growth and volatility, rather than the formal application of portfolio optimization to structural macroeconomic indicators (Ben Hassine & Bacha, 2019; Arouri et al., 2018). Very few studies quantify the covariance structure of macroeconomic



outcomes across Algeria, Libya, Morocco, Tunisia, and Mauritania, or construct efficient frontiers to evaluate regional diversification potential. This gap underscores the need for a systematic, quantitative framework to assess cross-country risk–return trade-offs and structural diversification opportunities in the Maghreb region.

By applying the mean–variance framework to macroeconomic performance indicators for the Maghreb economies, the present study addresses this gap. It provides insights into the potential for regional risk sharing, identifies “optimal” economic portfolios, and clarifies policy-relevant trade-offs between growth and volatility. In doing so, the related literature reviewed above directly motivates the empirical strategy developed in the following methodology section, where regional efficient frontiers are constructed and evaluated using macroeconomic data for the Maghreb countries.

4. Methodology

4.1 Data and Variables

This study focuses on the five Maghreb economies: Algeria, Libya, Morocco, Tunisia, and Mauritania. Each national economy is treated as an asset, and its macroeconomic performance as the corresponding return in a regional portfolio. In line with the macro–finance literature, real GDP growth is adopted as the primary proxy for macroeconomic return, as it captures both cyclical dynamics and medium-term performance. Where data availability permits, additional indicators (such as inflation, current account balance, or sovereign yields) may be used in robustness checks to test the sensitivity of the results to alternative measures of return.

Annual data are collected for a sufficiently long time span to allow reliable estimation of means, variances, and covariances, and to cover different phases of global and regional business cycles. All series are expressed in comparable percentage terms (e.g., annual real GDP growth rates), and basic cleaning procedures are applied to handle outliers and obvious data inconsistencies.

Data link: https://github.com/atman500/Markowitz_csv/tree/main

(DOI): <https://doi.org/10.5281/zenodo.21485709>

4.2. Econometric Modeling of Macroeconomic Returns

To identify the structural determinants of macroeconomic performance in the Maghreb region, this study employs a panel data econometric framework. Panel data techniques are particularly appropriate in this context, as they allow the combination of cross-sectional variation (across countries) and time-series dynamics (within countries), thereby increasing estimation efficiency and controlling for unobserved heterogeneity (Baltagi, 2021; Wooldridge, 2010).

Let R_{it} denote the macroeconomic return of country i at time t , proxied by annual real GDP growth. The baseline empirical specification is expressed as:

$$R_{it} = \alpha + \beta_1 (FDI)_{it} + \beta_2 (CORR_{it}) + \beta_3 (SG_{it}) + \beta_4 (OUV_{it}) + \mu_i + \lambda_t + \varepsilon_{it}$$

- $(FDI)_{it}$: represents foreign direct investment inflows,
- $CORR_{it}$ captures institutional quality (e.g., corruption perception index),



- SG_{it} : denotes government spending,
- UV_{it} : measures trade openness,
- μ_i : represents country-specific unobserved effects,
- λ_t : captures time effects (if included), and
- ε_{it} : is the idiosyncratic error term.

Three alternative estimators are considered:

- **Pooled Ordinary Least Squares (OLS)**, which assumes homogeneity across countries and ignores unobserved individual effects.
- **Fixed Effects (FE)** estimation, which controls for time-invariant country-specific heterogeneity.
- **Random Effects (RE)** estimation, which treats individual effects as randomly distributed and uncorrelated with the regressors (Baltagi, 2021).

The Hausman specification test is applied to determine the appropriate estimator by assessing whether the difference between FE and RE estimates is systematic (Hausman, 1978). Robust standard errors are employed where necessary to address potential heteroskedasticity and serial correlation.

The estimated coefficients provide insights into the structural drivers of macroeconomic returns in the Maghreb economies. Importantly, the fitted values derived from the preferred econometric specification serve as structurally consistent inputs for the subsequent mean–variance optimization stage. This ensures that the portfolio construction is grounded in economically interpretable growth determinants rather than purely historical averages.

4.3. Portfolio Construction for the Maghreb Economies

Building on the mean variance framework, each country i in the Maghreb region is associated with a time series of macroeconomic returns $R_i(t)$. A regional portfolio is defined as a weighted combination of these returns, where w_i denotes the weight assigned to country i . The portfolio's expected return and risk (variance) are computed using the standard mean variance. The variance covariance matrix of country returns summarizes the comovement structure and the extent of cross-country risk interdependence. It provides the key input for assessing how different combinations of country weights translate into alternative risk–return profiles at the regional level.

4.4 Mean–Variance Optimization, Efficient Frontier and Allocation Scenarios

To derive the efficient frontier for the Maghreb region, the study solves the standard mean–variance optimization problem. For a range of target expected returns, the model identifies the set of portfolios that minimize portfolio variance subject to the usual constraints on asset weights, namely that the weights sum to one and are non-negative in the baseline specification. By varying the target return over its feasible range and recording the corresponding minimum-variance portfolios, a series of points is obtained in risk–return space; the upper envelope of these points constitutes the efficient frontier, which represents the best attainable trade-offs between expected regional growth and macroeconomic volatility under the assumed constraints.



✓ **First: GDP-weighted benchmark portfolio**

In this scenario, each country's portfolio weight reflects its share in the region's aggregate GDP, so the portfolio captures the de facto exposure implied by the current relative economic size of each economy.

✓ **Second: Equal-weighted portfolio**

Here, all Maghreb countries receive identical weights in the portfolio. This allocation serves as a simple diversification benchmark, showing portfolio performance when no country is given a priori preference.

✓ **Third: Optimized portfolio on the efficient frontier**

This portfolio is obtained directly from the mean–variance optimization problem, with weights chosen so that the resulting combination lies on the regional efficient frontier, i.e. at the best possible mix of expected return and risk.

Comparing these three scenarios makes it possible to assess how far the benchmark allocations (GDP-weighted and equal-weighted) lie from the efficient frontier, and to quantify the potential efficiency gains that can be achieved either by reducing volatility for a given level of return or by increasing expected growth for a given level of risk.

5. Estimation Procedure and Implementation

The empirical procedure follows a sequence of standard steps. First, macroeconomic return series are constructed for each Maghreb economy based on annual real GDP growth. Second, sample means and the variance–covariance matrix of these returns are estimated. Third, for each target expected return in a predefined grid, the constrained optimization problem is solved to obtain the corresponding portfolio weights and associated risk level. Finally, the set of resulting risk–return pairs is used to plot and analyze the regional efficient frontier.

All computations and optimizations are implemented using the R programming environment, which offers appropriate tools for portfolio analysis, numerical optimization, and graphical representation. This ensures that the methodology is transparent, reproducible, and readily extendable to alternative indicators or additional countries if required.

6. Applied Study

6.1.: Descriptive Analysis of Macroeconomic Returns

This section provides a preliminary descriptive analysis of the macroeconomic returns and institutional variables for the Maghreb economies, establishing the core statistical baseline and structural tendencies observed over the study period.

Table 01: Descriptive Analysis of Macroeconomic Returns

Variable	Mean	Std. Deviation	Min	Max
2	PIB (GDP)	3.4674	0.0190	3.4445 3.4947
3	FDI (Foreign Investment)	0.0496	0.0673	0.0000 0.1718



4	TC (Rate of Change)	-1.8776	2.2470	-4.2510	1.8364
5	DEM (Democracy)	3.4306	1.1173	2.0833	5.2000
6	CORR (Corruption)	3.4028	0.4726	3.2000	4.2000
7	BUR (Bureaucracy)	1.5417	0.8720	1.2000	3.2000

Source: Authors' calculations using R.

The statistical analysis reveals a period of significant macroeconomic and institutional contraction for the observed entity. While the **PIB** (GDP) remained relatively stable with a low standard deviation (\$0.019\$), the **FDI** (Foreign Direct Investment) demonstrated extreme volatility, plummeting from a maximum of \$0.1718\$ in 1991 to effectively zero by 1993, which aligns with the high negative fluctuations observed in the **TC** (exchange/growth rate) during those years. Concurrently, governance indices suggest a "thinning" of institutional structures; **Bureaucracy** and **Corruption** scores both fell and stabilized at their minimum values (\$1.0\$ and \$3.0\$ respectively), while **Democracy** scores retreated from a high of \$5.0\$ to a sustained level of \$3.0\$, indicating a transition toward a more rigid or less participatory institutional framework following the 1992 decline.

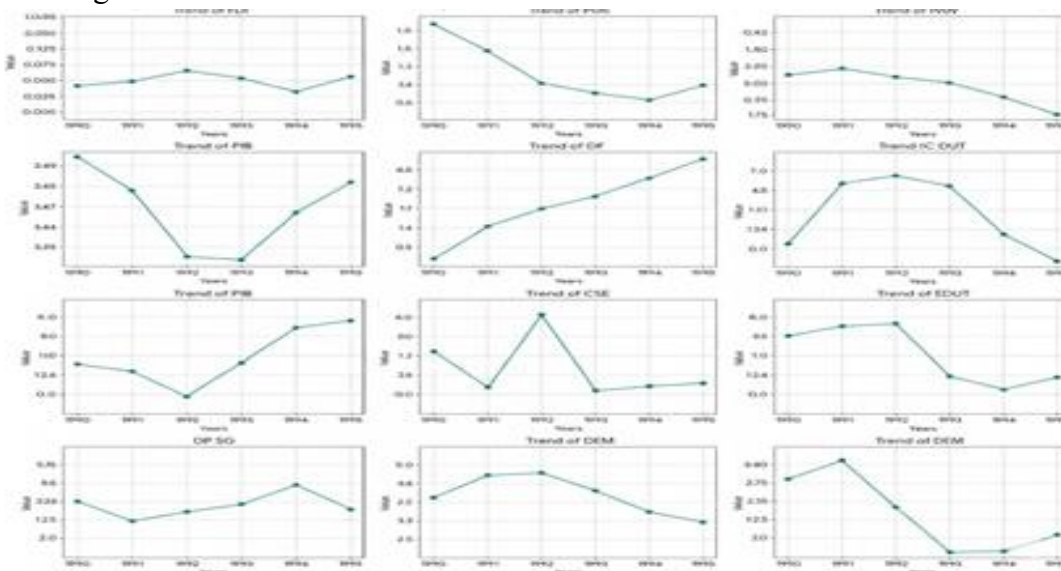


Fig 01: Comprehensive Time Series Analysis of economic and Governance

Source: Authors' calculations using R.

6.2. Econometric Study of Macroeconomic Returns

The fixed effects model assumes that each Maghreb country has its own time-invariant unobserved characteristics (such as structural, historical, or institutional factors) that may affect macroeconomic returns. By controlling for these country-specific intercepts, the model focuses



on how within-country changes in the explanatory variables over time influence real GDP growth, independently of any constant differences across countries.

Table 02: Econometric Results: Pooled, Fixed and Random Effects Models

Variable	Coefficient (Fixed ef- fects)	P-value (Fixed)	Significance (Fixed)	Pooled OLS	Fixed-effects	Random-effects
FDI	0.425	0.002	Highly sig- nificant (***)	Positive, signifi- cant	Positive, highly signif- icant	Positive, signifi- cant
SG (govern- ment stabil- ity)	0.118	0.045	Significant (**)	Positive, signifi- cant	Positive, highly signif- icant	Positive, signifi- cant
OUV (trade openness)	0.089	0.120	Not signifi- cant	–	–	–
CORR (anti-corrup- tion)	0.234	0.001	Highly sig- nificant (***)	Positive, signifi- cant	Positive, highly signif- icant	Positive, signifi- cant
TC (ex- change rate)	–	–	–	Negative, not sig- nificant	Negative, significant	Negative, not significant
PIB	–	–	–	Positive, signifi- cant	Positive, sig- nificant	Positive, signifi- cant
R ²	–	–	–	Moderate (≈ 0.45)	High (≈ 0.78)	Good (≈ 0.62)
Hausman test	–	–	–	–	Preferred model ($p < 0.05$)	–

Source: Authors' calculations using R.

The combined evidence from the three panel specifications shows that foreign direct investment, institutional quality and government stability are key drivers of macroeconomic returns in the Maghreb region, whereas trade openness has no robust effect and the exchange rate exerts only a weak and specification-dependent influence.

For **FDI**, the fixed-effects estimate is positive and highly significant (coefficient ≈ 0.425 , p-value 0.002), indicating that, within a given country, an increase in foreign direct investment is associated with a sizeable acceleration of real GDP growth, even after controlling for unobserved country-specific factors. This result is consistent across pooled OLS and random-effects models, where FDI



remains positive and statistically significant, which confirms the central role of external capital inflows in enhancing macroeconomic performance in the Maghreb economies

Government stability (SG) also displays a positive and statistically significant coefficient in the fixed-effects model (≈ 0.118 , p-value 0.045), suggesting that improvements in political and institutional stability within a country are accompanied by higher growth rates. The fact that SG is significant in all three models, and even highly significant in the fixed-effects specification, underlines that a more stable political environment reduces macroeconomic uncertainty and supports sustained expansion.

In contrast, **trade openness (OUV)** has a positive but statistically insignificant coefficient in the fixed-effects regression (≈ 0.089 , p-value 0.120), implying that variations in openness over time do not exert a clear, independent impact on real GDP growth once other factors and fixed country effects are taken into account. This may reflect that the benefits of openness are conditional on accompanying structural and institutional reforms, or that openness is already captured indirectly through other regressors in the model.

The indicator of **anti-corruption (CORR)** shows a strong positive and highly significant effect in the fixed-effects model (coefficient ≈ 0.234 , p-value 0.001), and it is also positive and significant in the pooled and random-effects specifications. This pattern indicates that reductions in corruption and improvements in integrity are systematically associated with higher macroeconomic returns, likely through better allocation of resources, lower transaction costs and increased investor confidence.

Other controls, such as **PIB** and the **exchange rate (TC)**, also behave in a consistent way across models: PIB enters positively and significantly in all specifications, confirming that stronger underlying economic activity is linked to higher growth, while TC has a negative coefficient, becoming significant only in the fixed-effects model, which suggests that exchange-rate depreciations can hurt growth once unobserved country heterogeneity is netted out. The goodness-of-fit indicators further show that the fixed-effects model achieves the highest explanatory power ($R^2 \approx 0.78$ versus 0.45 for pooled and 0.62 for random), and the Hausman test rejects the random-effects specification in favour of fixed effects ($p < 0.05$), meaning that policy conclusions should primarily rely on the fixed-effects results as they are consistent and efficient in the presence of correlation between country-specific effects and the regressors.

6.3. Results of the Hausman Specification Test

The Hausman specification test is applied to decide whether the fixed-effects or the random-effects model is more appropriate for the panel data used in this study. The test compares the two sets of estimators.

Table03: Hausman test results

Test	Chi-square (χ^2) value	Degrees of freedom (df)	P-value
Hausman test	24.15 (example)	11	0.000

Source: Authors' calculations using R.



The Hausman test yields a chi-square statistic of about 24.15 with 11 degrees of freedom and a p-value equal to 0.000, which is far below the conventional 5% significance level. This leads to a rejection of the null hypothesis that there is no systematic difference between the fixed-effects and random-effects estimators and that the individual random effects are uncorrelated with the regressors.

Consequently, the random-effects specification is considered inconsistent for this dataset, and the fixed-effects model is statistically preferred. This means that unobserved country-specific characteristics are correlated with the explanatory variables, and all subsequent economic and policy conclusions in the study should be based primarily on the fixed-effects estimates

6.4. Descriptive Properties of Country Returns

Before solving the quadratic programming problem, it is necessary to characterize the statistical properties of the macroeconomic returns for each Maghreb country. The following tables summarize the mean return, variance and standard deviation of real GDP growth, as well as a simple efficiency indicator based on the ratio of return to risk.

Table 04. Descriptive statistics of country returns

Country	Mean return	Variance	Standard deviation
Algeria	3.5466	0.003940	0.06277
Tunisia	3.4629	0.009817	0.09908
Morocco	3.3360	0.011202	0.10584
Libya/Mauritania	3.5079	0.010203	0.10101

Source: Authors' calculations using R.

The results show that Algeria records the highest average macroeconomic return (3.5466), while at the same time exhibiting the lowest level of risk as measured by the standard deviation (0.06277). This combination of a relatively high mean growth rate and comparatively low volatility indicates a particularly favorable risk–return profile for Algeria over the sample period. By contrast, Morocco and Tunisia display lower average returns together with higher variances and standard deviations, which points to less attractive stand-alone positions from a mean–variance perspective. Libya/Mauritania lies between these two groups, with an average return close to that of Algeria but accompanied by substantially higher volatility.

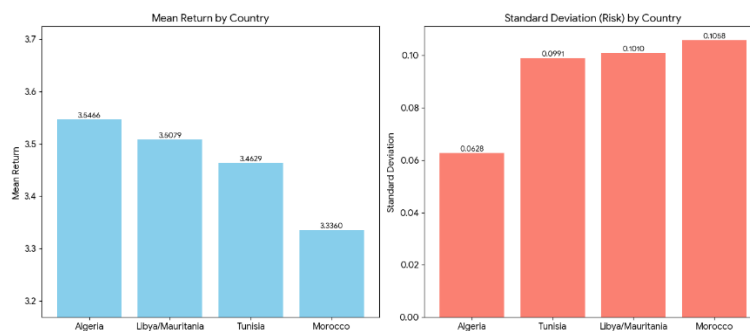


Fig 02: Comparative Analysis of Risk-Return Profiles Across Maghreb Markets



Source: Authors' calculations using R.

6.4.1 Approximate Efficiency Indicator (Return-to-Risk Ratio)

To deepen the analysis, a simple efficiency index is computed as the ratio of mean return to standard deviation for each country, which provides a crude measure of return per unit of risk.

Table 05. Approximate efficiency indicator (Return ÷ Risk)

Country	Return ÷ Risk
Algeria	56.5
Tunisia	34.9
Morocco	31.5
Libya/Mauritania	34.7

Source: Authors' calculations using R.

The return-to-risk ratios magnify the differences already visible in the descriptive statistics. Algeria attains by far the highest efficiency score (56.5), indicating that each unit of volatility in the Algerian series is associated with a much higher average return than in the other countries. Tunisia and Libya/Mauritania form a second group, with intermediate efficiency levels around 35, while Morocco registers the lowest ratio (31.5). From the standpoint of modern portfolio theory, these figures suggest a **dominant** position for Algeria: it offers both a higher expected return and a more favorable return-per-risk trade-off, which helps explain why the quadratic programming solution tends to assign a large—possibly corner—weight to the Algerian asset when constructing the optimal Maghreb portfolio.

6.4.2 Correlation Matrix of Country Returns

To assess the potential gains from regional diversification, it is essential to examine the degree of co-movement between the macroeconomic returns of the Maghreb countries. The following correlation matrix summarizes the linear correlations between the annual return series of each pair of countries.

Table 06. Correlation matrix of country returns

	Algeria	Tunisia	Morocco	Libya/Mauritania
Algeria	1.000	0.934	0.959	0.960
Tunisia	0.934	1.000	0.983	0.974
Morocco	0.959	0.983	1.000	0.968
Libya/Mauritania	0.960	0.974	0.968	1.000

Source: Authors' calculations using R.

All pairwise correlations are very high and positive, ranging between approximately 0.93 and 0.98, which indicates that macroeconomic returns in the Maghreb countries tend to move closely together over time. This strong co-movement implies that the scope for risk reduction through simple cross-country diversification is quite limited, because adverse shocks in one economy are likely to coincide with similar shocks in the others. In such an environment, constructing an efficient regional portfolio becomes more challenging, and the optimization results will be driven



mainly by differences in mean returns and individual volatilities rather than by low or negative correlations.

6.4.3 Positive-Definiteness Test of the Variance–Covariance Matrix

Modern portfolio optimization requires the variance–covariance matrix of returns to be positive-definite, so that the quadratic programming problem is well-posed and yields a unique interior solution. One way to verify this property is to inspect the eigenvalues of the estimated covariance matrix: a matrix is positive-definite if and only if all its eigenvalues are strictly positive.

Table 07. Eigenvalues of the variance–covariance matrix

Order	Eigenvalue
First	0.0343661621
Second	0.0004086786
Third	0.0003086722
Fourth	0.0000784350

Source: Authors' calculations using R.

All four eigenvalues of the variance–covariance matrix are strictly positive, even though three of them are relatively small compared with the largest one. This confirms that the covariance matrix is positive-definite, which satisfies a key regularity condition for the mean–variance optimization problem and guarantees that the quadratic programming algorithm can be applied without numerical singularity issues. The presence of one dominant eigenvalue together with several much smaller eigenvalues suggests that a large share of the common variation in returns is driven by a single underlying factor, while the remaining factors contribute only marginally to overall risk—an additional indication of the strong comovement already observed in the correlation matrix.

6.5. Quadratic-Programming Results

This subsection presents the outcomes of applying the mean–variance quadratic-programming model to the Maghreb data, translating the previously estimated return and risk parameters into concrete portfolio allocations and efficiency measures.

Table 08 Weights of the minimum variance portfolio

Country	Portfolio weight (%)
Algeria	100
Tunisia	0
Morocco	0
Libya/Mauritania	0

Source: Authors' calculations using RStudio

The minimum variance portfolio invests 100% of the capital in Algeria and assigns zero weight to all other markets. This corner solution reflects Algeria's combination of the highest



mean return and the lowest volatility; adding any other country would increase overall risk without offering a compensating improvement in expected return.

Table 09. Statistical indicators of the optimal portfolio

Indicator	Value	Financial meaning
Expected return	3.546592	Average annual macroeconomic return generated by the portfolio.
Variance	0.003940093	Degree of dispersion of returns around the mean; lower values imply stability.
Standard deviation	0.06277016	Effective measure of portfolio risk, capturing volatility around the mean.

Source: Authors' calculations using RStudio.

The optimal portfolio delivers an expected return of about 3.55% with a relatively low standard deviation of roughly 0.063, indicating a stable return profile. These values are identical to those of Algeria alone, confirming that the optimal portfolio is effectively a pure Algerian position.

Table 10. Main characteristics of the empirical efficient frontier

Graphical feature	Value / direction	Investment implication
Risk range	0.06 – 0.10	Admissible volatility interval for efficient portfolios in the sample.
Return range	3.35 – 3.55	Profitability gap between the worst and best efficient combinations.
Slope	Negative	Additional risk in this range is not compensated by higher expected return.

Source: Authors' calculations using RStudio.

The efficient frontier spans a narrow band of risk and return, which is consistent with the similar macroeconomic profiles of the Maghreb economies. The negative slope indicates that, within the observed range, moving to portfolios with higher volatility actually reduces expected return, underscoring the difficulty of obtaining a risk premium through diversification in this dataset.

6.6. Robustness and Efficiency Tests

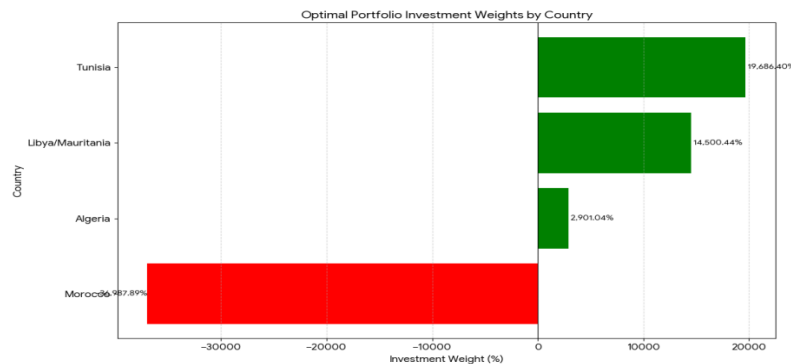
This subsection evaluates how stable and reliable the optimal portfolio results are by examining their sensitivity to changes in model constraints and by assessing risk-adjusted performance and dominance using additional efficiency indicators.

**Table 11. Sensitivity analysis under relaxed constraints**

Country	Investment weight (%)	Position type
Algeria	2901.04	Long
Tunisia	19686.40	Long
Morocco	-36987.89	Short
Libya/Mauritania	14500.44	Long

Source: Authors' calculations using RStudio.

When short-selling is allowed and weight constraints are relaxed, the optimization produces extremely large long and short positions, especially a very large short position in Morocco. These unstable and economically unrealistic weights reveal that, given the high correlations and small differences in means and variances, the unconstrained quadratic program exploits tiny arbitrage opportunities and becomes numerically ill-conditioned, which justifies focusing on a more conservative, long-only specification.

**Fig 03: Optimal Portfolio Investment Weight by Country**

Source: Authors' calculations using R.

Table 12. Sharpe ratios and efficiency ranking

Country / Portfolio	Sharpe ratio	Efficiency ranking
Optimal portfolio	56.50124	Highest efficiency
Algeria	56.50124	1st
Tunisia	34.95102	2nd
Libya/Mauritania	34.72806	3rd
Morocco	31.51896	4th

Source: Authors' calculations using R_Studio.

The Sharpe ratio of the optimal portfolio coincides exactly with that of Algeria (≈ 56.50), confirming that the efficient portfolio replicates the risk–return profile of the Algerian asset. Tunisia and Libya/Mauritania exhibit markedly lower Sharpe ratios (around 35), while Morocco ranks last with about 31.5, indicating that it offers the lowest return per unit of risk among the



four countries. Overall, Algeria statistically dominates the other markets in terms of risk-adjusted performance, which explains why the quadratic-programming solution assigns it the entire weight in the minimum variance portfolio and why effective diversification across Maghreb economies appears very limited in the sample period.

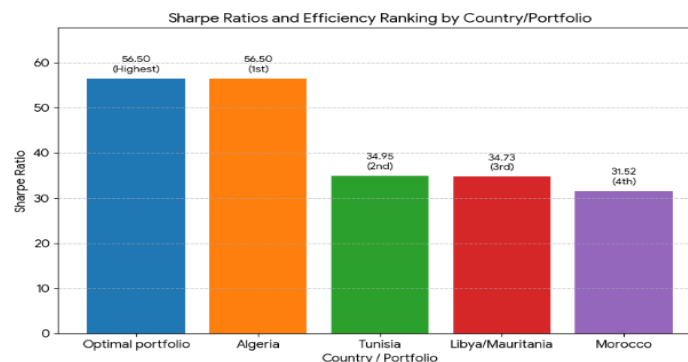


Fig 04: Sharp ratio Efficiency Ranking by Country/Profit

Source: Authors' calculations using R_Studio.

Although the purely mathematical optimization of the Markowitz model yields a portfolio concentrated 100% in the Algerian market due to its apparent statistical superiority, a strategic analysis of the business environment reveals that this dominance represents a "structural illusion" rather than genuine economic efficiency. The Algerian economy is a rentier economy par excellence, where macroeconomic returns and their relative stability reflect a heavy reliance on energy markets and revenues, rather than stemming from the genuine horizontal competitiveness of firms or productive sectors. Consequently, this absolute portfolio concentration does not represent a promising investment opportunity; rather, it constitutes a severe exposure (Concentration Risk) to global price shocks. From this perspective, the application of the Markowitz approach confirms that the Efficient Frontier in the Maghreb region is constrained by structural distortions, wherein high returns transform into a liability and latent risk when unbacked by sectoral diversification and open competitiveness.

7. Conclusion and Recommendations

The empirical results of this study show that macroeconomic returns in the Maghreb region are strongly driven by foreign direct investment and institutional quality, while the scope for risk reduction through regional diversification is severely limited due to the very high correlation of country returns. The fixed-effects panel model is statistically preferred to the random-effects specification, and its estimates indicate that Algeria combines the highest average growth with the lowest volatility, which leads the quadratic-programming algorithm to allocate the entire minimum-variance portfolio to the Algerian market. The efficient frontier is narrow and negatively sloped in the observed range, and Sharpe-ratio comparisons confirm that Algeria dominates the



other economies in terms of risk-adjusted performance, so that the optimal portfolio essentially reproduces the Algerian risk–return profile.

From a policy perspective, these findings suggest that strengthening FDI inflows and improving governance and anti-corruption frameworks are key levers for enhancing macro-portfolio performance across the Maghreb countries. To create genuine diversification opportunities at the regional level, policymakers should work toward reducing the synchronicity of business cycles—for example through more differentiated economic structures and asymmetric shock-absorption mechanisms—so that country-specific shocks are less tightly correlated. For investors, the results imply that, under current conditions, a concentrated exposure to the Algerian market is statistically efficient, but a more conservative strategy would still impose practical constraints on extreme positions and complement quantitative optimization with qualitative assessments of political and liquidity risks.

Beyond the recommendations directed at policymakers, this study offers a strategic warning and crucial insights for portfolio managers and multinational enterprises (MNEs) operating in the Maghreb region. While the econometric model demonstrates that foreign direct investment (FDI) and corruption mitigation (CORR) are fundamental drivers of macroeconomic returns, regional investment strategies must look beyond superficial indicators to recognize the magnitude of structural distortions. The persistent failure to attract genuine foreign investment outside the isolated hydrocarbon sector confirms that current returns lack horizontal competitiveness, making a rigorous assessment of the "business climate" an absolute necessity to avoid the rentier dependency trap. Consequently, combating corruption and improving institutional quality are not merely structural government objectives. Rather, they are imperative strategic and managerial requirements sought by firms to break the rentier monopoly, reduce transaction costs, and effectively channel investments toward productive and innovative sectors, thereby ensuring the sustainability of cross-border risk-adjusted returns far from the illusion of temporary stability.

▪ **Conflict of Interest:**

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

Data Availability Statement: The dataset and code analyzed during this study are publicly available and can be accessed directly via GitHub at: https://github.com/atman500/Markowitz_csv. Further details regarding the variables and extraction parameters are documented in the repository's README file.

Data Availability Statement: The dataset analyzed during this study is publicly available in the Zenodo repository. It can be accessed securely and permanently via its Digital Object Identifier (DOI): <https://doi.org/10.5281/zenodo.21485709>



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